LA DISTRIBUZIONE NORMALE (CURVA DI GAUSS)

DISTRIBUZIONE NORMALE o DISTRIBUZIONE DI GAUSS

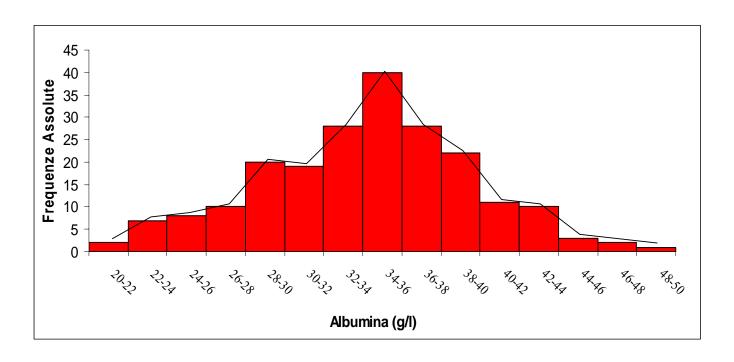
- 1. E' la più importante distribuzione continua e trova numerose applicazioni nello studio dei fenomeni biologici.
- 2. Fu proposta da Gauss (1809) nell'ambito della teoria degli errori, ed è stata attribuita anche a Laplace (1812), che ne definì le proprietà principali in anticipo rispetto alla trattazione più completa di Gauss.
- 3. Il nome "normale" deriva dalla convinzione che molti fenomeni fisicobiologici si distribuiscono con frequenze più elevate nei valori centrali e frequenze progressivamente minori verso gli estremi.
- 4. E' detta anche CURVA DEGLI ERRORI ACCIDENTALI in quanto, soprattutto nelle discipline fisiche, la distribuzione degli errori commessi nel misurare ripetutamente la stessa grandezza, è molto bene approssimata da questa curva.

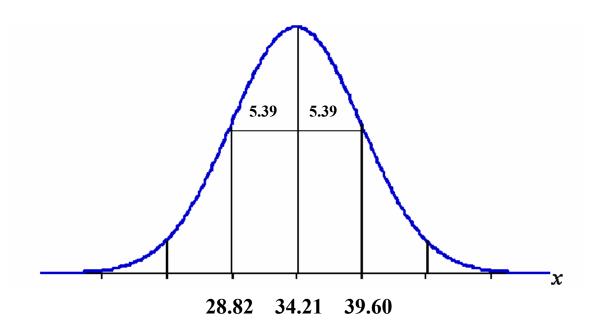
Esempio 1 - Distribuzione di 211 pts affetti da cirrosi biliare primitiva rispetto ai valori della concentrazione sierica di albumina

Valori albumina (g/l)	Frequenze	Percentuali	Percentuali cumulate
20- 22	2	0,95	0,95
22-24	7	3,32	4,27
24-26	8	3,79	8,06
26-28	10	4,74	12,80
28-30	20	9,48	22,27
30-32	19	9,00	31,28
32-34	28	13,27	44,55
34-36	40	18,96	63,51
36-38	28	13,27	76,78
38-40	22	10,43	87,20
40-42	11	5,21	92,42
42-44	10	4,74	97,16
44-46	3	1,42	98,58
46-48	2	0,95	99,53
48- 50	1	0,47	100,00
Totale	211	100,00	

MEDIA ARITMETICA (μ)= 34.21 g/l DEVIAZIONE STANDARD (σ) = 5.39 g/l

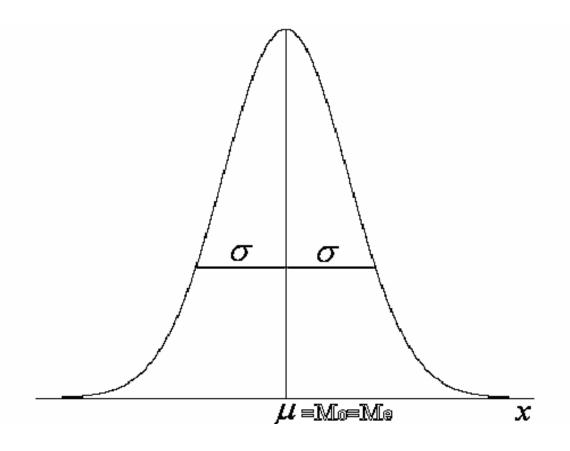
Figura 1 - Istogramma di frequenza della concentrazione sierica di albumina in 211 pazienti con cirrosi biliare primitiva



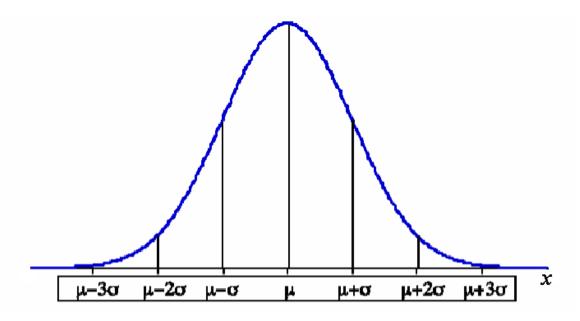


Quando le distribuzioni di frequenza prima risultano crescenti, raggiungono un massimo e poi cominciano a decrescere fino ad arrivare allo zero, si può parlare di variabili che tendono a distribuirsi "normalmente", ossia che seguono un andamento secondo la curva di Gauss la cui espressione analitica è del tipo:

$$y_i = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{\sum (x_i - \mu)^2 f_i}{2\sigma^2}\right)$$

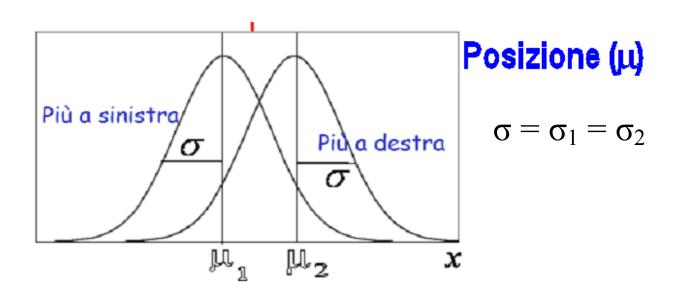


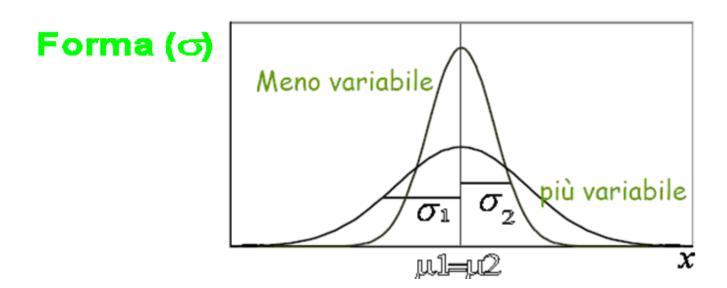
CARATTERISTICHE DELLA DISTRIBUZIONE NORMALE



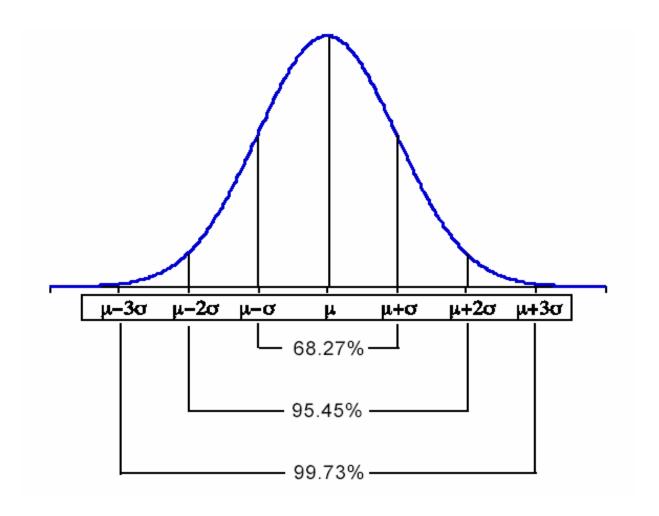
- 1. è simmetrica rispetto al valore medio
- 2. il valore di $x = \mu$ oltre che alla media aritmetica coincide anche con la moda e la mediana
- 3. è asintotica all'asse delle x da entrambi i lati
- 4. è crescente per $x < \mu$ e decrescente per $x > \mu$
- 5. possiede due punti di flesso per $x = \mu \pm \sigma$
- 6. l'area sotto la curva $ext{è} = 1$ (essendo la probabilità che si verifichi un qualsiasi valore di x)

I parametri caratterizzanti una Distribuzione Normale ne caratterizzano la POSIZIONE e la FORMA





INTERVALLI NOTI DI PROBABILITÀ



N:B: In realtà sono μ±1.96σ e μ±2.98σ

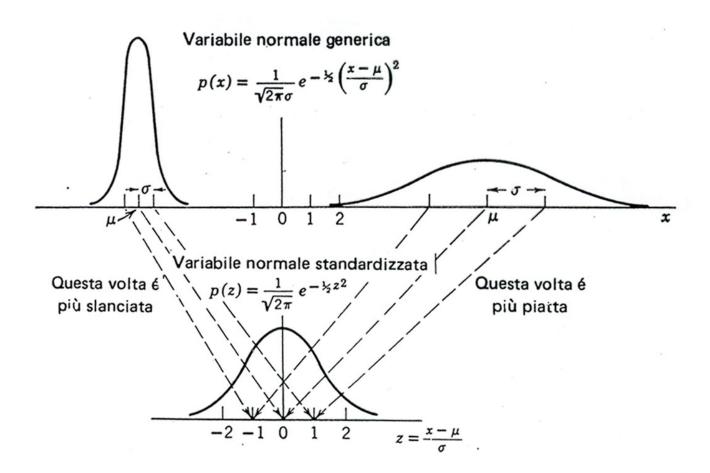
Tornando all'esempio 1:

Valori albumina (g/l)	Frequenze	Percentuali	Percentuali cumulate
20 -22	2	0,95	0,95
22-24	7	3,32	4,27
24-26	8	3,79	8,06
26-28	10	4,74	12,80
28-30	20	9,48	22,27
30-32	19	9,00	31,28
32-34	28	13,27	44,55
34-36	40	18,96	63,51
36-38	28	13,27	76,78
38-40	22	10,43	87,20
40-42	11	5,21	92,42
42-44	10	4,74	97,16
44-46	3	1,42	98,58
46-48	2	0,95	99,53
48-50	1	0,47	100,00
Totale	211	100,00	

$$\mu \pm \sigma = (28.8-39.6) = 64.93\%$$
 $\mu \pm 2\sigma = (23.4-44.9) = 94.31\%$
 $\mu \pm 3\sigma = (18.0-50.4) = 100\%$

DISTRIBUZIONE NORMALE STANDARDIZZATA

Una distribuzione Normale che ha media 1 e DS 0 è chiamata distribuzione normale standardizzata.



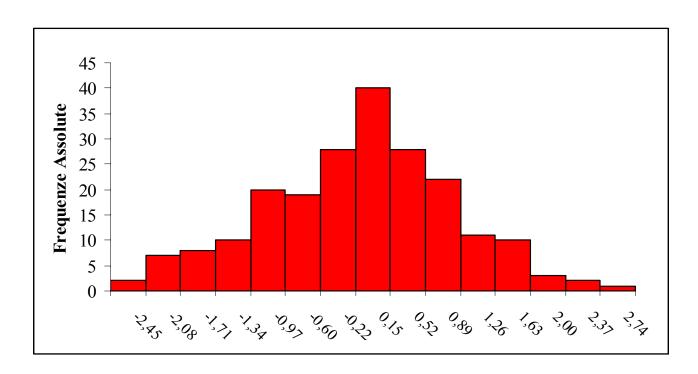
La variabile standardizzata è definita attraverso la trasformazione

$$Z = \frac{X - \mu}{\sigma}$$

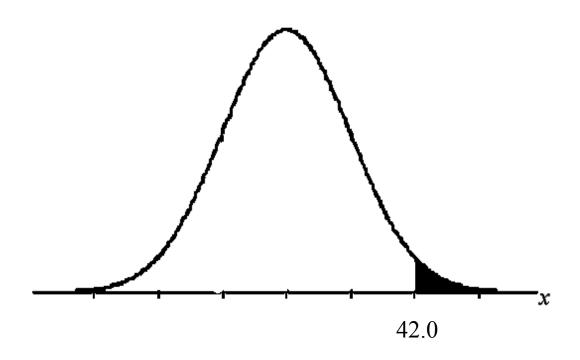
Valor	i albumina			Percentuali cumulate	
Grezzi (X)	Standardizzati (Z)	Frequenze	Percentuali		
21	-2.45	2	0,95	0,95	
23	-2.08	7	3,32	4,27	
25	-1.71	8	3,79	8,06	
27	-1.34	10	4,74	12,80	
29	-0.97	20	9,48	22,27	
31	-0.60	19	9,00	31,28	
33	-0.22	28	13,27	44,55	
35	0.15	40	18,96	63,51	
37	0.52	28	13,27	76,78	
39	0.89	22	10,43	87,20	
41	1.26	11	5,21	92,42	
43	1.63	10	4,74	97,16	
45	2.00	3	1,42	98,58	
47	2.37	2	0,95	99,53	
49	2.74	1	0,47	100,00	
Totale	Totale Totale		100,00		

 $MEDIA\ ARITMETICA = 0$

DEVIAZIONE STANDARD = 1



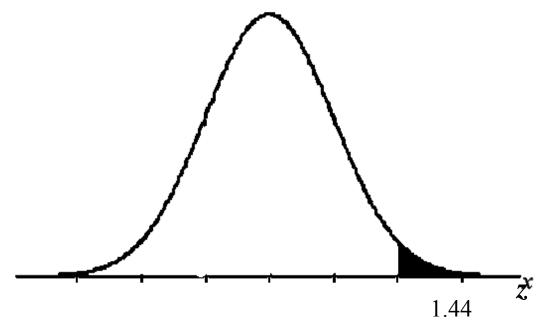
Ora ci chiediamo: qual è la probabilità per un paziente affetto da cirrosi biliare primitiva di avere un valore di albumina ≥42.0 g/l?
Cioè, qual' è la P(X≥42.0)



Trasformando il valore di 42.0 g/l in valori di Z

$$Z=(X-\mu)/\sigma=(42.00-34.21)/5.39=1.44$$

Dall'esempio 1 (pag. 3)



 $P(Z \ge 1.44) = 1-0.9251 = 0.0749 = 7.49\%$

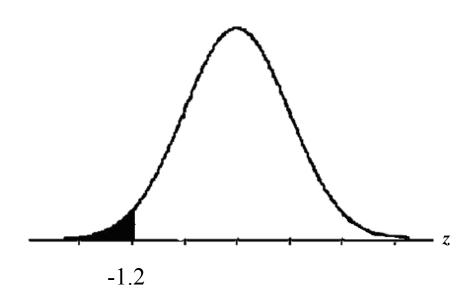
Un paziente con albumina > 42 g/l ha il 7.5% di probabilità di essere affetto da cirrosi, e il 92.5% dei pazienti cirrotici presenta un valore < 42 g/l

Esempio 1. Dalla letteratura scientifica risulta che in una popolazione (P) apparentemente sana il valore medio dell'HDL-colesterolo è di 57 mg/100ml e DS 10 mg/100ml.

Sapendo che i valori dell'HDL (high density lipoprotein) si distribuiscono normalmente, si vuole stimare la probabilità che un soggetto appartenente alla P abbia valori di HDL <45 mg/100ml.

$$Z = (45-57)/10 = -1.2$$

$$P(Z<-1.2) = 1 - 0.8849 = 0.1151 = 11.51\%$$



La percentuale di soggetti con valori di HDL<45 mg/100ml nella popolazione è del 11.51%.

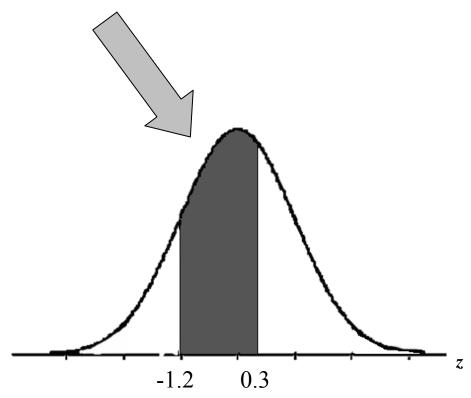
Si vuole stimare la probabilità che un soggetto abbia valori di HDL compresi tra 45 mg/100 ml e 60 mg/100ml. Mediante la standardizzazione si ha:

$$Z_1 = (45-57)/10 = -1.2$$

$$Z_2 = (60-57)/10 = 0.3$$

$$P(-1.2 < Z < 0.3) = P(Z < 0.3) - [1 - P(Z < 1.2)] = 0.6179 - [1 - 0.8849] =$$

$$0.1151 = 0.5028 = 50.28\%$$



NORMALITA' STATISTICA



DISTRIBUZIONE NORMALE

NORMALITA' DIAGNOSTICA:

Come valutare normale o patologico un segno clinico o un esame di laboratorio?

Tale problema può essere affrontato solo in termini di probabilità, con tecniche statistiche.

La misurazione di un carattere biologico in una popolazione dà in ciascun individuo risultati diversi indipendentemente dalla VARIABILITA' DEL CARATTERE e dalla IMPRECISIONE SPERIMENTALE delle tecniche di misura.

Se il carattere si distribuisce nella popolazione NORMALMENTE i valori di normalità, al 95% di probabilità, "cadono" nell'intervallo:

$$\overline{x} - 2DS$$
 $\overline{x} + 2DS$

Esempio 2. Da dati ufficiali rilevati sulla popolazione nazionale risulta che il valore medio della Glicemia è di 92 mg/dl e DS 20 mg/dl.

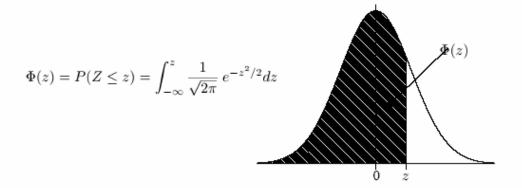
L'intervallo di normalità è dunque 92±2*20 mg/dl.

Si ha una probabilità del 5% di compiere un errore dichiarando "patologico" un valore all'esterno dell'intervallo 52-132 mg/dl.

In realtà solo i soggetti con valori di Glicemia >132 mg/dl sono patologici.

Non sempre la normalità "cade" in un intervallo a volte è per valori > di o <
di.

Tavola 1: Funzione di ripartizione della Variabile Casuale Normale Standardizzata



z 0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.0 0.5000 0.5040 0.5080 0.5120 0.5100 0.5199 0.5239 0.5279 0.5319 0.5359 0.2 0.5793 0.5832 0.5871 0.5517 0.5557 0.5596 0.5636 0.5675 0.5714 0.5753 0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064 0.6103 0.6141 0.3 0.6179 0.6217 0.6255 0.6293 0.6331 0.6368 0.6406 0.6443 0.6480 0.6614 0.5 0.6915 0.6950 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.5 0.6915 0.6905 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.6 0.7257 0.7291 0.7357											
0.1 0.5398 0.5438 0.5478 0.5517 0.5557 0.5596 0.5636 0.5675 0.5714 0.5753 0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6044 0.6103 0.6117 0.4 0.6554 0.6591 0.6628 0.6664 0.6700 0.6736 0.6772 0.6808 0.6844 0.6879 0.5 0.6915 0.6950 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.6 0.7257 0.7291 0.7324 0.7337 0.7389 0.7422 0.7454 0.7486 0.7517 0.7549 0.7 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7744 0.7745 0.7450 0.8210 0.8113 0.8106 0.8133 0.9080 0.8101 0.	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.2 0.5793 0.5832 0.5871 0.5910 0.5948 0.5987 0.6026 0.6064 0.6103 0.6114 0.3 0.6179 0.6217 0.6255 0.6293 0.6331 0.6368 0.6406 0.6443 0.6480 0.6517 0.4 0.6554 0.6591 0.6628 0.6664 0.6700 0.6736 0.6772 0.6808 0.6844 0.6879 0.5 0.6915 0.6905 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.6 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7549 0.7 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7764 0.7794 0.7823 0.7823 0.7823 0.7825 0.8023 0.8051 0.8078 0.8136 0.8212 0.8238 0.8264 0.8289 0.8315 0.8343 0.8665 0.8483 0.8461 0.	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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0.4 0.6554 0.6591 0.6628 0.6664 0.6700 0.6736 0.6772 0.6808 0.6844 0.6879 0.5 0.6915 0.6950 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.6 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7764 0.7753 0.7852 0.8813 0.8063 0.8810 0.8810 0.8810 0.8810 0.8810 0.8812 0.	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.5 0.6915 0.6950 0.6985 0.7019 0.7054 0.7088 0.7123 0.7157 0.7190 0.7224 0.6 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7549 0.7 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7764 0.7794 0.7823 0.7852 0.8 0.7881 0.7910 0.7939 0.7967 0.7995 0.8023 0.8051 0.8078 0.8106 0.8133 0.9 0.8159 0.8186 0.8212 0.8238 0.8264 0.8289 0.8315 0.8344 0.8365 0.8389 1.0 0.8413 0.8485 0.8666 0.8708 0.8729 0.8770 0.8790 0.8810 0.8830 1.2 0.8849 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.99049 0.9916 0.9922	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.6 0.7257 0.7291 0.7324 0.7357 0.7389 0.7422 0.7454 0.7486 0.7517 0.7549 0.7 0.7580 0.7611 0.7642 0.7673 0.7704 0.7734 0.7794 0.7823 0.7852 0.8 0.7881 0.7910 0.7939 0.7967 0.7995 0.8023 0.8051 0.8078 0.8106 0.8133 0.9 0.8159 0.8186 0.8212 0.8238 0.8264 0.8289 0.8315 0.8340 0.8365 0.8381 1.0 0.8413 0.8465 0.8686 0.8708 0.8729 0.8749 0.8770 0.8790 0.8810 0.8830 1.2 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9117 0.9147 0.9162 0.9177 1.4 0.9192 0.9222 0.9236	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
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0.8 0.7881 0.7910 0.7939 0.7967 0.7995 0.8023 0.8051 0.8078 0.8106 0.8133 0.9 0.8159 0.8186 0.8212 0.8238 0.8264 0.8289 0.8315 0.8340 0.8365 0.8389 1.0 0.8413 0.8438 0.8461 0.8485 0.8508 0.8531 0.8570 0.8579 0.8621 1.1 0.8643 0.8665 0.8686 0.8708 0.8729 0.8770 0.8790 0.8810 0.8830 1.2 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177 1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370	0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
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1.0 0.8413 0.8438 0.8461 0.8485 0.8508 0.8531 0.8554 0.8577 0.8599 0.8621 1.1 0.8643 0.8665 0.8686 0.8708 0.8729 0.8749 0.8770 0.8790 0.8810 0.8830 1.2 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177 1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9533 0.9546 1.7 0.9544 0.9640	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
1.1 0.8643 0.8665 0.8686 0.8708 0.8729 0.8749 0.8770 0.8790 0.8810 0.8830 1.2 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177 1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9534 1.7 0.9554 0.9564 0.9573 0.9582 0.9590 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656	0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.2 0.8849 0.8869 0.8888 0.8907 0.8925 0.8944 0.8962 0.8980 0.8997 0.9015 1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177 1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9531 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9756 0.9761 0.9767 2.0 0.9772 0.9778 0.9788 0.9793 0.9798 0.9803 0.9812 0.9817 2.1 <t< td=""><td>1.0</td><td>0.8413</td><td>0.8438</td><td>0.8461</td><td>0.8485</td><td>0.8508</td><td>0.8531</td><td>0.8554</td><td>0.8577</td><td>0.8599</td><td>0.8621</td></t<>	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.3 0.9032 0.9049 0.9066 0.9082 0.9099 0.9115 0.9131 0.9147 0.9162 0.9177 1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9545 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9508 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9698 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793	1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.4 0.9192 0.9207 0.9222 0.9236 0.9251 0.9265 0.9279 0.9292 0.9306 0.9319 1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9545 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9804 0.9812 0.9817 2.1 0.9821 0.9864 0.9868	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.5 0.9332 0.9345 0.9357 0.9370 0.9382 0.9394 0.9406 0.9418 0.9429 0.9441 1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9545 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 2.2 0.9861 0.9868 0.9871	1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515 0.9525 0.9535 0.9545 1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9756 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9857 0.9857 2.2 0.9861 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9933 0.9896 0.9898	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.7 0.9554 0.9564 0.9573 0.9582 0.9591 0.9599 0.9608 0.9616 0.9625 0.9633 1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9756 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9857 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9913 0.9932 0.9934 0.9936 2.5	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.8 0.9641 0.9649 0.9656 0.9664 0.9671 0.9678 0.9686 0.9693 0.9699 0.9706 1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9756 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9884 0.9887 0.9880 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9913 0.9932 0.9936 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9935 2.5 0.9938	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.9 0.9713 0.9719 0.9726 0.9732 0.9738 0.9744 0.9750 0.9756 0.9761 0.9767 2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9913 0.9913 0.9913 0.9914 0.9916 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 <td>1.7</td> <td>0.9554</td> <td>0.9564</td> <td>0.9573</td> <td>0.9582</td> <td>0.9591</td> <td>0.9599</td> <td>0.9608</td> <td>0.9616</td> <td>0.9625</td> <td>0.9633</td>	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
2.0 0.9772 0.9778 0.9783 0.9788 0.9793 0.9798 0.9803 0.9808 0.9812 0.9817 2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9916 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9974 2.8 0.9974 0.9975 0.9976	1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
2.1 0.9821 0.9826 0.9830 0.9834 0.9838 0.9842 0.9846 0.9850 0.9854 0.9857 2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9916 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9940 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9973 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9985 0.9985 0.9986 0.9986	1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.2 0.9861 0.9864 0.9868 0.9871 0.9875 0.9878 0.9881 0.9884 0.9887 0.9890 2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9916 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9940 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9985 0.9980 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.3 0.9893 0.9896 0.9898 0.9901 0.9904 0.9906 0.9909 0.9911 0.9913 0.9916 2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9940 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9980 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9999 0.9999	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.4 0.9918 0.9920 0.9922 0.9925 0.9927 0.9929 0.9931 0.9932 0.9934 0.9936 2.5 0.9938 0.9940 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9989 0.9989 0.9989 0.9989 0.9999 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9994 0.9994 0.9994 0.9994 0.9994 0.9996	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.5 0.9938 0.9940 0.9941 0.9943 0.9945 0.9946 0.9948 0.9949 0.9951 0.9952 2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989 0.9999 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9994 0.9992 0.9992 0.9992 0.9992 0.9995 0.9995 0.9995 3.2 0.9993 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.6 0.9953 0.9955 0.9956 0.9957 0.9959 0.9960 0.9961 0.9962 0.9963 0.9964 2.7 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9992 0.9995 0.9995 0.9996 0.	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.7 0.9965 0.9966 0.9967 0.9968 0.9969 0.9970 0.9971 0.9972 0.9973 0.9974 2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 2.9 0.9981 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9989 0.9989 0.9989 0.9999 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9992 0.9993 0.9993 0.9995 0.9995 3.2 0.9995 0.9995 0.9996	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.8 0.9974 0.9975 0.9976 0.9977 0.9977 0.9978 0.9979 0.9979 0.9980 0.9981 2.9 0.9981 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9993 0.9993 0.9993 3.2 0.9993 0.9994 0.9994 0.9994 0.9994 0.9994 0.9994 0.9996 0.99	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.9 0.9981 0.9982 0.9982 0.9983 0.9984 0.9984 0.9985 0.9985 0.9986 0.9986 3.0 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9992 0.9993 0.9993 3.2 0.9993 0.9994 0.9994 0.9994 0.9994 0.9994 0.9994 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
3.0 0.9987 0.9987 0.9987 0.9988 0.9988 0.9989 0.9989 0.9989 0.9990 0.9990 3.1 0.9990 0.9991 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9993 0.9993 0.9993 3.2 0.9993 0.9993 0.9994 0.9994 0.9994 0.9994 0.9994 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
3.1 0.9990 0.9991 0.9991 0.9991 0.9992 0.9992 0.9992 0.9992 0.9993 0.9993 0.9993 3.2 0.9993 0.9994 0.9994 0.9994 0.9994 0.9994 0.9995 0.9995 0.9995 3.3 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996	2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.2 0.9993 0.9993 0.9994 0.9994 0.9994 0.9994 0.9995 0.9995 0.9995 3.3 0.9995 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996 0.9996 0.9997	3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.3 0.9995 0.9995 0.9995 0.9996 0.9996 0.9996 0.9996 0.9996 0.9997	3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
	3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.4 0.9997 0.9997 0.9997 0.9997 0.9997 0.9997 0.9997 0.9997 0.9998	3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
	3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998